

IC/95/349
hep-th/9510164
October 23, 1995

SUPERSYMMETRY BREAKING IN ORBIFOLD COMPACTIFICATIONS

KARIM BENAKLI*

International Centre of Theoretical Physics, Trieste, Italy

ABSTRACT

Known mechanisms for breaking of supersymmetry at the level of string theory imply that at least one of the internal dimensions has a very large size. Experimental detection of the associated light Kaluza-Klein (KK) excitations would be a strong hint for the existence of string like elementary objects, as no consistent field theory describing them is known. We restrict the discussion to the Scherk-Schwarz mechanism in orbifold compactifications. For this case we investigate the quantum number of the lightest predicted KK states.

Talk presented at the
Workshop on strings, gravity and related topics
Trieste, Italy, June 29 – 30, 1995

*E-mail: benakli@ictp.trieste.it

Supersymmetry appears quite naturally in superstring theory. However understanding its breaking remains an open problem. If this breaking is due to non-perturbative effects then it can not be studied directly at the level of string models within the actual perturbative formulation. Another possibility is that supersymmetry is broken at tree level. This is the case of supersymmetry breaking by a magnetic field¹ or through the string version of the Scherk-Schwarz mechanism². In both case one finds that the gravitino or gauginos get masses inversely proportional to the size of some internal dimension. This is in agreement with the result that a small supersymmetry breaking scale implies a large internal dimension³. Here we review the case of the Scherk-Schwarz mechanism where the fermions and bosons have mass splitting due to different compactification boundary conditions⁴.

The simplest framework to study supersymmetry breaking through the Scherk-Schwarz mechanism are the orbifold compactifications⁵. These are four-dimensional string models with $N = 1$ space-time supersymmetry obtained from toroidal compactification by dividing out some discrete subgroup of the automorphisms of the Hilbert space. Here the elements of the discrete subgroup are combinations of translation shifts and rotations. The resulting physical Hilbert space consists in twisted and untwisted sectors. The twisted sectors contain states that don't have internal momenta so, at the tree level, they don't feel the supersymmetry breaking mechanism. The untwisted sector is obtained from the Hilbert space of a string propagating on a torus by projecting on invariant states under the action of the orbifold. The mass spectrum in this sector is determined by the associated internal momenta through⁶⁻⁹:

$$\frac{1}{4}m_L^2 = \frac{1}{4}m_R^2 = N_R + \frac{1}{2}\mathbf{p}_R^2, \quad (1)$$

where N_R is the oscillator number and \mathbf{p}_R is the internal momentum given by:

$$p_R = (m_i - a_i^I(p^I - \frac{1}{2}a_j^I n^j) + \xi_{ij}^* Q^j - \frac{\xi_{ki}\xi_{kj}^*}{2}n^j)\frac{\mathbf{e}^{*i}}{2R_i} - n^i R_i \mathbf{e}_i. \quad (2)$$

In the above formula p^I is the gauge internal momentum, m_i are the momenta number, n^i are winding number. a_i^I are the Wilson lines and Q^j is the charge that takes integer and fractional values for the bosons and fermions respectively, breaking supersymmetry. The requirement that the orbifold projection and gauge symmetry breaking commute imposes a condition on the allowed Wilson lines^{5,10} and reduces the maximum number of independent discrete Wilson lines. The parameters ξ_{ij}^* take discrete values and they parameterize the Lorentz boost which takes the theory from the unbroken supersymmetric phase to the broken one. Moreover the gauge symmetry breaking is also achieved through a Lorentz boost^{6,11}. The combination of the both Wilson lines and Scherk-Schwarz charge is then equivalent to a boost on the vector $(Q^A, p^I, p_L^a; p_R^{a'})$ given by⁹:

$$\begin{pmatrix} \delta_{AB} & 0 & -\frac{1}{2}\xi_{Ab} & \frac{1}{2}\xi_{Ab'} \\ 0 & \delta_J^I & \frac{1}{2}A_b^I & -\frac{1}{2}A_{b'}^I \\ \frac{1}{2}\xi_{aB}^* & -\frac{1}{2}A_a^J & \delta_{ab} - \frac{1}{8}\xi_{Ca}\xi_{Cb}^* - \frac{1}{8}A_a^K A_b^K & \frac{1}{8}\xi_{Ca}\xi_{Cb'}^* + \frac{1}{8}A_a^K A_{b'}^K \\ \frac{1}{2}\xi_{a'B}^* & -\frac{1}{2}A_{a'}^J & -\frac{1}{8}\xi_{Ca'}\xi_{Cb}^* - \frac{1}{8}A_{a'}^K A_b^K & \delta_{a'b'} + \frac{1}{8}\xi_{Ca'}\xi_{Cb'}^* + \frac{1}{8}A_{a'}^K A_{b'}^K \end{pmatrix} \quad (3)$$

which leads to the spectrum (2).

It is important to notice that the breaking of gauge symmetry and supersymmetry commute as they are two similar (but different) Lorentz boosts. Then there is no new condition imposed on the charge Q^A . That allows us to study the properties of the models in their supersymmetric phase.

The requirement that orbifold projection, gauge symmetry and supersymmetry breaking commute, restricts the allowed Wilson lines and the charges Q^A . The charge Q^A can be written as: $Q^A = \oint J^A$ where J^A is a $U(1)$ current which shouldn't commute with the $2d$ supercurrent so that it gives a charge for the gravitino but not to the graviton and gauge bosons which are usually in the untwisted sector.

In orbifold compactifications, the condition that Q^A is associated with some particular direction A means that it should have the same transformation under the orbifold group than the corresponding coordinate ∂X^A . This requirement is very strong as it leaves only few possible $U(1)$ currents.

Different currents were found for the cases of Z_N and $Z_N \times Z_M$ orbifolds⁹. For the orbifolds \mathbf{Z}_4 and $\mathbf{Z}_2 \times \mathbf{Z}_2$ the $U(1)$ charges were already known⁷. We have focused on orbifolds where only one dimension (\mathbf{Z}_2 case) or two dimensions are large. For example, this excludes \mathbf{Z}_7 orbifold which needs the six internal dimensions to be of the same size. For the orbifolds \mathbf{Z}_3 , \mathbf{Z}_6 , \mathbf{Z}_8 and \mathbf{Z}_{12} as well as $\mathbf{Z}_2 \times \mathbf{Z}_6$, $\mathbf{Z}_3 \times \mathbf{Z}_3$, $\mathbf{Z}_3 \times \mathbf{Z}_6$, $\mathbf{Z}_4 \times \mathbf{Z}_4$ and $\mathbf{Z}_6 \times \mathbf{Z}_6$ we have not found charges allowing to implement the Scherk-Schwarz mechanism.

Such theories with perturbative breaking of supersymmetry have become recently of some phenomenological interest after it has been shown that they could allow a weakly coupled string theory, at least at one-loop for a class of models based on orbifold compactifications⁷. We have found only two orbifolds \mathbf{Z}_4 and $\mathbf{Z}'_6 \equiv \mathbf{Z}_2 \times \mathbf{Z}_3$ which have charges associated with $N = 4$ sectors where the possible large threshold corrections vanish. The other orbifolds lead to light KK-states in $N = 2$ multiplets. In this case, the one loop threshold depending on the value of the large radius is not automatically vanishing. One would have then to chose the particle content as KK-excitations to get vanishing β -functions⁷.

In these theories the manifestation of the large extra-dimension(s) would be the existence of KK-excitations that would appear as some new particles with regularly spaced masses and behaving as excitations of the MSSM particles. In the limit where some supersymmetry (thus electroweak) breaking effects are neglected, some properties as the quantum numbers and interactions of these states in orbifold compactifications have been investigated¹². The viability of these theories requires that⁸:

i) As the fermions from the untwisted sector acquire a common mass-shift, the quarks and leptons must be identified with twisted states. This rules out all the string $SU(3) \times SU(2) \times U(1)$ models build in the past.

ii) If the Higgs doublets appear in the untwisted sector, the generation of masses for untwisted fermions allows for generating the μ - term.

It has been pointed out¹² that if the untwisted sector is ‘minimal’ the only observable effects of the KK-excitations are through some non-renormalizable effective operators. The latter have then been computed and limits on the size of new dimensions have been derived from actual experimental data¹². The obtained bounds allow the hope of experimental detection in the near future¹³. They could also have some cosmological implications¹⁴.

An obvious question to address is the compatibility of these requirements with orbifold compactifications. In other words, what are the new light states we expect in realistic models?. Notice first that in the gauge symmetry breaking process, the states acquire masses inversely proportional to the radius of the torus corresponding to the Wilson line. The massless states (in the supersymmetric phase) can easily be seen to correspond to Wilson lines singlets: $aP \in \mathbf{Z}$. While we see that a Wilson line associated to the torus with the large radius used to break supersymmetry will lead to a mass of the order of hundreds GeV or TeV to the projected states. In particular, if some states have $0 < |aP| < 1$ as it is often the case, then the corresponding states will have masses smaller than the KK excitations of the states, with different gauge quantum numbers, present at the massless level. This also implies that the minimal light KK states are obtained when all the Wilson lines have to be associated only to the other small tori. Such a minimally requirement would also automatically avoid the presence of some massive new vector bosons that could mediate new dangerous interactions.

The formula (2) shows that all the states carrying the same gauge internal momenta have the same masses. In particular, this implies that all the $N = 2$ and $N = 4$ multiplets get projected by the gauge symmetry breaking and only $N = 4$ (or $N = 2$) excitations of massless untwisted states are present among the light KK states.

We have also to deal with the effect of reducing the rank of the gauge group on the Kaluza-Klein excitations. The Higgs mechanism through discrete Wilson lines described above doesn’t reduce the rank of the gauge group. To reduce the rank one can embed the Wilson lines in the gauge group as automorphism of the Γ_{16} lattice¹⁵. This corresponds to the case where the orbifold action on the gauge lattice is described by a rotation $\Theta \neq 0$. In this case some Cartan generators of the gauge group are not associated with a root of Γ_{16} , but with an invariant combination of winding states. In the case where some components of the Wilson line are rotated by Θ , the projection on Wilson line singlets projects out, in general, the Cartan generators which have the form of invariant combination of winding states. As this projection is at the level of the gauge lattice state, which is the same for all the KK excitations of the gauge boson, all the KK tower is projected out. Both the rank of the gauge symmetry group and the rank of the symmetry group of the KK

excitations are reduced simultaneously. Notice that if the Wilson line is associated with the dimension with large size then the projected states are very light.

We have investigated¹⁶ the minimal light untwisted states obtained from \mathbf{Z}_4 and $\mathbf{Z}'_6 \equiv \mathbf{Z}_2 \times \mathbf{Z}_3$ orbifolds. For \mathbf{Z}_4 we found that from $E_8 \times E_8$, the minimal untwisted spectrum is the adjoint representation of $SU(2) \times SU(4) \times U(1) \times \dots$ or $SU(4) \times SU(4) \times U(1) \times \dots$ if we want Higgs like doublets from the untwisted sector. From $Spin(32)/Z_2$ we can obtain $SU(3) \times SU(3) \times U(1) \times \dots$ by using for example the following Wilson lines:

$$a_1 = \frac{1}{4} (1, 1, -1, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1); \quad (4)$$

$$a_2 = \frac{1}{4} (0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0); \quad (5)$$

$$a_3 = \frac{1}{4} (2, 2, 0, 0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0); \quad (6)$$

but it is hard to find corresponding orbifold shifts leading to chiral quark doublets in the spectrum. For \mathbf{Z}'_6 we found that from $E_8 \times E_8$ we can get $SU(3) \times SU(3) \times U(1) \times \dots$ but as for the first case it is (at least) difficult to get three generation models.

Many questions regarding the proposed mechanism for breaking SUSY still remain open. However one of the nice features of this scenario is that it makes precise predictions that could be tested at future colliders.

Acknowledgements

I wish to thank I. Antoniadis, E. Gava, K.S. Narain, M. Quiròs, J. Rizos and A. Sen for discussing different parts of the material presented here.

References

1. C. Bachas, preprint CPTH-R349-0395 hep-th/9503030 (1995).
2. R. Rohm, *Nucl. Phys.* **B237** (1984) 553; C. Kounnas and M. Porrati, *Nucl. Phys.* **B310** (1988) 355; S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, *Nucl. Phys.* **B318** (1989) 75; C. Kounnas and B. Rostand, *Nucl. Phys.* **B341** (1990) 641.
3. R. Rohm and E. Witten, *Ann. Phys.* **170** (1986) 454; T. Banks and L. Dixon, *Nucl. Phys.* **B307** (1988) 93; I. Antoniadis, C. Bachas, D. Lewellen and T. Tomaras, *Phys. Lett.* **207B** (1988) 441; S.P. de Alwis, J. Polchinski and R. Schimmrigk, *Phys. Lett.* **218B** (1989) 449.
4. J. Scherk and J.H. Schwarz, *Phys. Lett.* **B82** (1979) 60 and *Nucl. Phys.* **B153** (1979) 61; E. Cremmer, J. Scherk and J.H. Schwarz, *Phys. Lett.* **B84** (1979) 83; P. Fayet, *Phys. Lett.* **B159** (1985) 121 and *Nucl. Phys.* **B263**

- (1986) 649; Proc. 2nd Nobel Symposium on El. Part. Physics at Marstrand (Sweden, 1986), Physica Scripta T15 (1987) 46
5. L.J. Dixon, J. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B261** (1985) 678; **B274** (1986) 285.
 6. K.S. Narain, *Phys. Lett.* **B169** (1986) 41; K.S. Narain, M.H. Sarmadi and E. Witten *Nucl. Phys.* **B279** (1987) 369; P. Ginsparg, *Phys. Rev.* **D35** (1987) 648.
 7. I. Antoniadis, *Phys. Lett.* **246B** (1990) 377; Proc. PASCOS-91 Symposium, Boston 1991 (World Scientific, Singapore) p.718.
 8. I. Antoniadis, C. Muñoz and M. Quirós, *Nucl. Phys.* **B397** (1993) 515.
 9. K. Benakli, preprint IC/95/306 hep-th/9509115 (1995).
 10. L.E. Ibáñez, H. P. Nilles, F. Quevedo, *Phys. Lett.* **187B** (1987) 25.
 11. S. Ferrara, C. Kounnas and M. Porrati, *Nucl. Phys.* **B304** (1988) 500; I. Antoniadis, C. Bachas and C. Kounnas, *Phys. Lett.* **200B** (1988) 297.
 12. I. Antoniadis and K. Benakli *Phys. Lett.* **326B** (1994) 69.
 13. I. Antoniadis, K. Benakli and M. Quirós *Phys. Lett.* **331B** (1994) 313.
 14. S. Abel and S. Sarkar *Phys. Lett.* **342B** (1995) 40.
 15. L.E. Ibáñez, H. P. Nilles, F. Quevedo, *Phys. Lett.* **192B** (1987) 332.
 16. K. Benakli, preprint in preparation.